

# A relativistic study of the nucleon form factors<sup>\*</sup>

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Received: 9 October 1997

Communicated by V.V. Anisovich

**Abstract.** We perform a calculation of the relativistic corrections to the electromagnetic elastic form factors of the nucleon obtained with various Constituent Quark Models. With respect to the non relativistic calculations a substantial improvement is obtained up to  $Q^2 \simeq 2(GeV/c)^2$ .

**PACS.** 12.39.Ki Relativistic quark model – 13.40.Gp Electromagnetic form factors – 14.20.Dh Protons and neutrons

## 1 Introduction

The non relativistic constituent quark models (CQM) have given good results in the study of the static properties of the nucleon [1,2], like the baryon spectrum and the magnetic moments, and in a qualitative reproduction of the photocouplings [3–5]. However, the standard CQ-Models are unable to reproduce the  $Q^2$  behaviour of the electromagnetic form factors even in the low momentum transfer [6–10].

The use of harmonic oscillator models give rise to form factors which decrease too fast with respect to the experimental data. Some improvement of this behaviour, specially in the case of the transition form factors, can be obtained by using more realistic wave functions [9,5]. However, the problem of a reasonable description of the elastic and transition form factors of the nucleon in the framework of a Constituent Quark model is still open.

The inclusion of relativistic effects is expected to be important in the description of the nucleon form factors. The structure of the electromagnetic current of a relativistic bound system still represents an unsolved problem. Much attention has been recently devoted to this problem, following substantially three main lines: the expansion of relativistic current operators in powers of the inverse quark mass,  $\frac{1}{m}$ , the evaluation of the current matrix elements in a light-cone approach and the expansion of the full relativistic current matrix elements, again in powers of  $\frac{1}{m}$ .

The first approach takes into account the relativistic effects in the electromagnetic operators [11,12], the baryon states being the standard CQM ones. This type of

relativistic correction includes the  $q\bar{q}$ -pair contribution to two-body currents coming from the one-gluon exchange [13,14]. The numerical results show that these effects are significant but not sufficient to explain the data.

There are many interesting results obtained in a light-cone approach [8,15,16], such as the fact that the relativistic corrections to the transition form factors are important at low  $Q^2$  [8] and that the root mean square radius of the proton is increased [8,16]. This method is very useful since it allows to perform calculations starting from non relativistic wave functions. However there are still some problems in extracting form factors from the evaluated current matrix elements.

In this work we follow the third method which consists of expanding the current matrix elements in powers of  $\frac{1}{m}$  [17,18] and we propose a simplified approach useful for a preliminary calculation of the relativistic corrections to the elastic electromagnetic form factors of the nucleon obtained starting from different Constituent Quark Models. The use of Lorentz boosts for the quark spinors ensures that the relation between the dynamic variables of the initial and final states is relativistically correct. On the other hand, we assume that the quark internal motion is well described by the standard non relativistic wave function. The current matrix elements are constructed with a quark current operator containing only one-body terms and no quark form factors are introduced. We point out that the non relativistic expansion of the matrix elements of the present work, up to order  $m^{-2}$ , is coincident with that given by standard procedures [20,19,18] introduced for the few-nucleon systems and no approximation is done with respect to the momentum transfer  $Q^2$  dependence.

In Sect. 2, we describe the evaluation of the current matrix elements arriving at simple analytical expressions for the form factors. In Sect. 3, we discuss the results obtained with various 3q-wave functions and make a com-

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<sup>\*</sup> Partially supported by EC-contract number ERB FMRX-CT96-0008

parison with the experimental data. A brief conclusion is given in Sect. 4.

## 2 The current matrix elements

For the study of the transition process between the initial (I) and final (F) states, we have to calculate the current matrix element

$$J_{FI}^\mu = \langle \bar{\Psi}_F | \sum_{i=1}^3 j^\mu(i) | \Psi_I \rangle, \quad (1)$$

where  $j^\mu(i)$  is the e.m. current of the  $i$ -th quark. We choose the Breit frame and so the total initial and final tetramomenta  $P_I = (E_I, \mathbf{P}_I)$ ,  $P_F = (E_F, \mathbf{P}_F)$  are related by

$$\mathbf{P}_I = -\mathbf{P}_F = -\frac{\mathbf{q}}{2}, \quad E_I = E_F = \sqrt{M^2 + \frac{\mathbf{q}^2}{4}} \equiv E \quad (2)$$

where  $\mathbf{q}$  is the virtual photon momentum,  $Q^2 = \mathbf{q}^2$  and  $M$  is the nucleon mass. We denote with  $p_i^*(i = 1, 2, 3)$  the quark tetramomenta in the nucleon rest frame and we introduce the relative three-momenta

$$\mathbf{p}_\rho = \frac{1}{\sqrt{2}}(\mathbf{p}_1^* - \mathbf{p}_2^*), \quad \mathbf{p}_\lambda = \frac{1}{\sqrt{6}}(\mathbf{p}_1^* + \mathbf{p}_2^* - 2\mathbf{p}_3^*) \quad (3)$$

which are conjugated to the standard Jacobi coordinates  $\boldsymbol{\rho}$  and  $\boldsymbol{\lambda}$ . The 3-quark state is assumed to be

$$\Psi_I = \prod_{i=1}^3 B_i u_i(p_i^*) \phi(\mathbf{p}_\rho, \mathbf{p}_\lambda), \quad (4)$$

where the  $B_i$  ( $i = 1, 2, 3$ ) are the usual Dirac boost operators that transform the quark spinors  $u_i(p_i^*)$  from the nucleon rest frame to the Breit one. The boosted spinors,  $\psi_i = B_i u_i(p_i^*)$ , have the covariant normalization

$$\bar{\psi}_i \psi_i = 1. \quad (5)$$

In (4)  $\phi(\mathbf{p}_\rho, \mathbf{p}_\lambda)$  is the standard non relativistic 3q-wave function, where for simplicity we have omitted the spin and isospin variables. The final state is written in a similar way. The current operator of the  $i$ -th quark,  $j_\mu(i)$ , has the form

$$j_\mu(i) = \sqrt{\frac{m}{\epsilon'_i}} \gamma_i^\mu \sqrt{\frac{m}{\epsilon_i}}, \quad (6)$$

where  $m$  is the quark mass and  $\epsilon_i$  ( $\epsilon'_i$ ) is the initial (final) quark energy in the Breit frame. The normalization factors  $\sqrt{\frac{m}{\epsilon'_i}}$ ,  $\sqrt{\frac{m}{\epsilon_i}}$ , have been introduced in order to obtain for the current matrix elements the correct expansion in powers of  $\frac{1}{m}$  (i.e. coincident with what is usually quoted in the literature) as shown in [20]. The quark energies  $\epsilon_i$ ,  $\epsilon'_i$  are then expressed in terms of the corresponding quantities in the nucleon rest frame by means of standard Lorentz transformations.

Finally, we add a factor  $2E$  to the matrix element of (1) in order to take into account the normalization of the total matrix element.

Because of the antisymmetry of the 3q-states, we can substitute  $\sum_{i=1}^3 j_\mu(i)$  with  $3j_\mu(3)$ . The interacting quark absorbs the photon threemomentum in the Breit frame and therefore taking into account the Lorentz boost on the 3-quark, we can write the momentum conservation as follows:

$$\mathbf{p}'_\lambda = \mathbf{p}_\lambda - \sqrt{\frac{2}{3}} \frac{M}{E} \mathbf{q}, \quad \mathbf{p}'_\rho = \mathbf{p}_\rho \quad (7)$$

where the apices refer to the final momenta. The resulting expression for the current matrix element is complicated because of the presence of non local terms coming from the momentum dependence and the calculation can be performed numerically. However, in order to arrive at a preliminary calculation of the relativistic corrections to the e.m. current, we introduce some simplified assumptions.

First, consistently with the use of a non relativistic model for the internal nucleon dynamics we approximate the quark energies in the nucleon rest frame as  $\epsilon_i^* \simeq m$ . Furthermore, we perform an expansion keeping contributions up to the first order in the relative quark momenta, but we treat exactly the dependence on the momentum transfer  $\mathbf{q}$ . To this end, we introduce in the current matrix element of (1) the variable  $\boldsymbol{\pi}_\lambda$  that is related to  $\mathbf{p}_\lambda$  and  $\mathbf{p}'_\lambda$  in the following way

$$\mathbf{p}'_\lambda = \boldsymbol{\pi}_\lambda - \frac{1}{2} \sqrt{\frac{2}{3}} \frac{M}{E} \mathbf{q} \quad (8)$$

$$\mathbf{p}_\lambda = \boldsymbol{\pi}_\lambda + \frac{1}{2} \sqrt{\frac{2}{3}} \frac{M}{E} \mathbf{q}. \quad (9)$$

From the previous equations one also has

$$\boldsymbol{\pi}_\lambda = \frac{1}{2}(\mathbf{p}'_\lambda + \mathbf{p}_\lambda). \quad (10)$$

We expand the current matrix element of (1) by keeping up to the linear terms in  $\boldsymbol{\pi}_\lambda$  and  $\mathbf{p}_\rho$ . In correspondence, the electric and magnetic form factors have zero- and first-order contributions.

The results for the zero-th order charge and magnetic form factors can be given in a simple analytical form

$$G_E^{(0)}(Q^2) = \frac{E}{M} (t_S)^2 t_I G_E^{nr}(Q^2 \frac{M^2}{E^2}) \quad (11)$$

$$G_M^{(0)}(Q^2) = \frac{E}{M} (t_S)^2 t_I \frac{g_\sigma}{2m} G_M^{nr}(Q^2 \frac{M^2}{E^2}), \quad (12)$$

where  $G_E^{nr}$ , and  $G_M^{nr}$  are the electric and magnetic form factors as given by the non relativistic quark model. The quantities  $t_S$ ,  $t_I$  and  $g_\sigma$

$$t_S = \frac{1}{Mm} [E\eta_S - \frac{M}{E} \frac{Q^2}{12}], \quad (13)$$

$$t_I = \frac{Mm}{E\eta_I + \frac{MQ^2}{6E}} \quad (14)$$

$$g_\sigma = \frac{2}{3} + \frac{\eta_I}{M} \quad (15)$$

with

$$\eta_S = \left[ m^2 + \frac{M^2}{36E^2} Q^2 \right]^{1/2}, \quad (16)$$

$$\eta_I = \left[ m^2 + \frac{M^2}{9E^2} Q^2 \right]^{1/2}, \quad (17)$$

as multiplicative factors.

The first order contribution to the charge density matrix element is essentially of spin-orbit nature and a non relativistic expansion up to order  $m^{-2}$  gives the sum of the standard and the anomalous spin-orbit terms. The first order term will be omitted in our calculations, since it gives a numerically negligible contribution for nucleon states which, according to the models we use, are mainly in S-wave. The first order corrections to the magnetic form factors are of two types, spin-orbit like and convective. The first one can be disregarded for the same reason quoted above for the charge form factor, while the convective part gives in any case a small contribution.

Therefore, within these approximations, the relativistic corrections introduce two kinds of modifications with respect to the non relativistic treatment: a multiplicative factor coming from the expansion of the quark spinors and the argument of the non relativistic form factors, i.e. the momentum transfer squared  $Q^2$ , being replaced by  $Q^2 \frac{M^2}{E^2}$ .

The current matrix elements must satisfy the current conservation equation

$$q_\mu J_{FI}^\mu = 0, \quad (18)$$

and it is satisfied in our case since the Constituent Quark Models we have used are based on local interactions.

### 3 Results and comparison with experimental data

The form factors of (11) and (12) can be calculated using as input the nucleon form factors obtained in a non relativistic quark model. We present the results for different choices of the quark interaction, namely the h.o. [1] (Fig. 1), the three-body force hypercentral potential [21] (Fig. 2) and an exactly solvable potential based on a hypercoulomb interaction [9] (Fig. 2). All these models have been used for the description of the spectrum [1, 21, 9] and of the photocouplings [3–5, 9]. The three-body force approach has allowed also a systematic analysis of the transition form factors for the excitation of the baryon resonances [10]. All of them contain also a spin dependent (hyperfine) interaction, which is essential for the description of the  $N - \Delta$  splitting and for the excitation of quite a few resonances. For the elastic form factors, the configuration mixing coming from the hyperfine interaction does not produce strong effects, apart from the neutron charge form factor, and we shall omit it here.

In the h.o. case, the choice of the h.o. parameter  $\alpha$  is crucial. There are many different values of  $\alpha$  quoted

in the literature, according to the quantities to be fitted [22]. We report in Fig. 1 the results obtained with 1)  $\alpha = 0.229 GeV$ , which gives the correct r.m.s. radius of the proton [1, 23] without the relativistic corrections, and 2)  $\alpha = 0.410 GeV$ , which is necessary in order to reproduce the photoexcitation of the  $D_{13}$  and  $F_{15}$  resonances [3, 4] and corresponds to a confinement radius of the order of  $0.5 fm$ . We note that the relativistic corrections increase the r.m.s. with respect to the non relativistic calculation [24]. In fact from (11) one gets, for the h.o. proton charge form factor,

$$\langle r^2 \rangle = \frac{1}{\alpha^2} + \frac{6}{M^2}, \quad (19)$$

where  $M$  is the nucleon mass and  $m = \frac{M}{3}$ . In order to get the correct radius one should use  $\alpha = 0.285 GeV$ , which however is not too different from the choice 1). The results of Fig. 1 show that the relativistic corrections improve the h.o. form factors, but the  $Q^2$  behaviour is still different from the experimental data.

In Fig. 2 we give the form factors obtained starting from the non relativistic calculation performed with the three-body force potential of [21]. This potential has the form  $V(x) = -\frac{\tau}{x} + b_{conf}x$ , where  $x$  is the hyperradius  $x = \sqrt{\rho^2 + \lambda^2}$  and the values of the parameters are  $\tau = 4.59$  and  $b_{conf} = 1.61 fm^{-2}$ . It should be noted that with this choice of the parameters and the inclusion of the standard hyperfine interaction, the three-body force allows to describe consistently the non-strange baryon spectrum [21], the photocouplings [5] and the electromagnetic transition form factors [10].

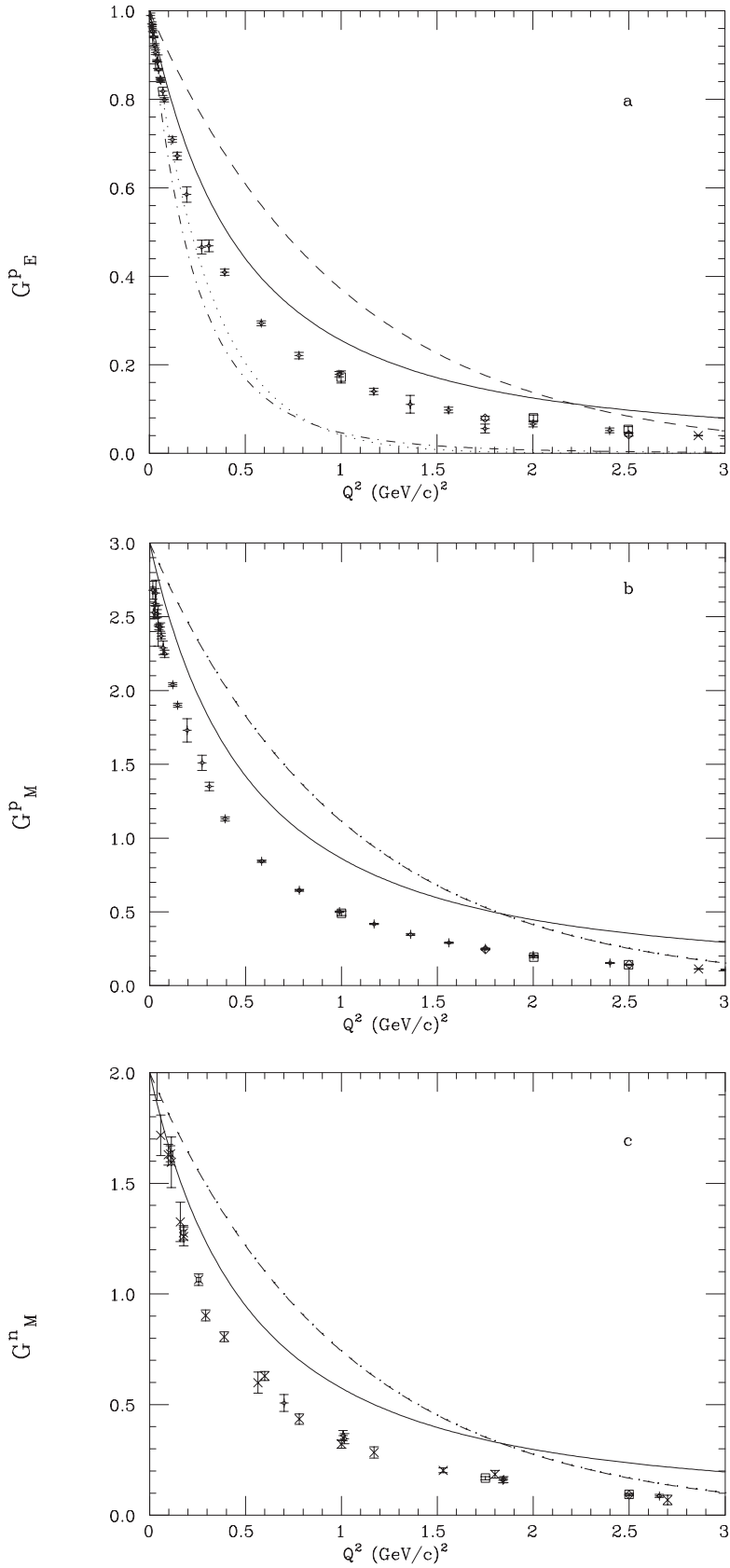
In Fig. 2 we give also the results for the solvable model of [9]. It is based on the hypercoulomb potential  $V_{hyc}(x) = -\frac{\tau}{x}$ , with  $\tau = 6.39$ , to which a small confinement term is added. The advantage of this potential is that the results can be given in analytical form. For instance, the proton charge form factor is given by

$$G_E^{nr}(Q^2) = \frac{1}{\left[1 + \frac{25}{24} \frac{Q^2}{\tau^2 m^2}\right]^{\frac{7}{2}}}. \quad (20)$$

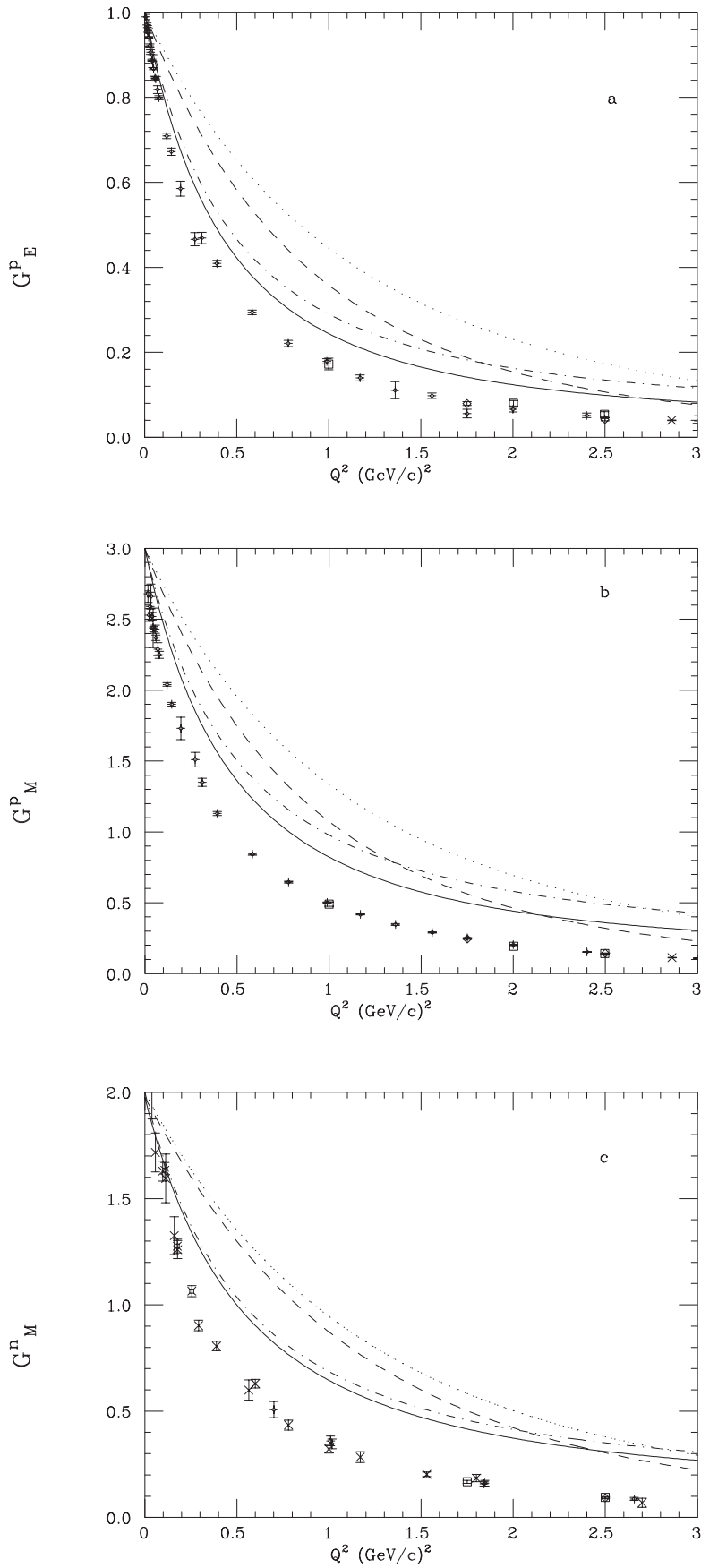
which, at variance with the h.o. case, for large  $Q^2$  has a power-law behaviour.

From the analysis of the results of Figs. 1 and 2, one sees that in general the inclusion of relativistic corrections improves significantly the non relativistic calculations. The improvement at low  $Q^2$  is related to the correct non relativistic limit of the current matrix elements. The improvement at higher  $Q^2$  depends on the relation between the initial and final state variables and allows to keep exactly the  $Q^2$  dependence of the form factors.

It should be noted that the simultaneous reproduction of the spectrum and the photocouplings requires a confinement radius of the order of  $0.5 fm$  [3, 4, 21, 5] and the relativistic increase is quite beneficial, but it is still not sufficient to get nearer to the data. In particular there still remain problems in the  $Q^2$  behaviour in the low and medium range. Similar problems are encountered also in the transition form factors, both in the relativistic [8] and in the non relativistic [10, 9] calculations, and so one can



**Fig. 1.** The charge **a** and magnetic **b** form factor of the proton and **c** the magnetic form factor of the neutron. The *curves* are the h.o. calculations using  $\alpha = 0.229$  GeV (*dotted* and *dot-dashed*) or  $\alpha = 0.410$  GeV (*dashed* and *full*). The *dashed* and *dotted* curves are the non relativistic calculations, the *full* and *dot-dashed* are the corresponding relativistic ones, obtained from (11) and (12). The experimental data are taken from the compilation of [15]



**Fig. 2.** The charge **a** and magnetic **b** form factor of the proton and **c** the magnetic form factor of the neutron. The *curves* obtained using the model of [21] are the non relativistic (*dashed*) and relativistic calculations (*full*). The *curves* obtained using the model of [9] are the non relativistic (*dotted*) and relativistic calculations (*dot-dashed*). The data are the same as in Fig. 1

think that not only the relativistic corrections are responsible for the discrepancies between the CQM calculations and the experimental data. As already noted elsewhere [10,9,6], some fundamental dynamical mechanism (effective at large distance, which means at low  $Q^2$ ) is still lacking, such as the explicit inclusion of quark-antiquark pairs both in the baryon states and in the electromagnetic transition operator.

## 4 Conclusions

We have calculated the relativistic corrections to the elastic nucleon form factors in a simplified and preliminary approach which leads to simple analytical expressions. We have used as input different Constituent Quark Models, namely the harmonic oscillator [1], the hypercentral model of [21] and the analytical model of [9] showing that in all the three models the relativistic corrections are important, since they bring the non relativistic calculations nearer to the data, but still they are not sufficient. The persisting discrepancy may be an indication that further degrees of freedom ( $q\bar{q}$ -pairs, gluons) should be included in the CQM in a more explicit way.

We gratefully thank Prof. D. Prosperi for many useful discussions and suggestions during the development of the present work.

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